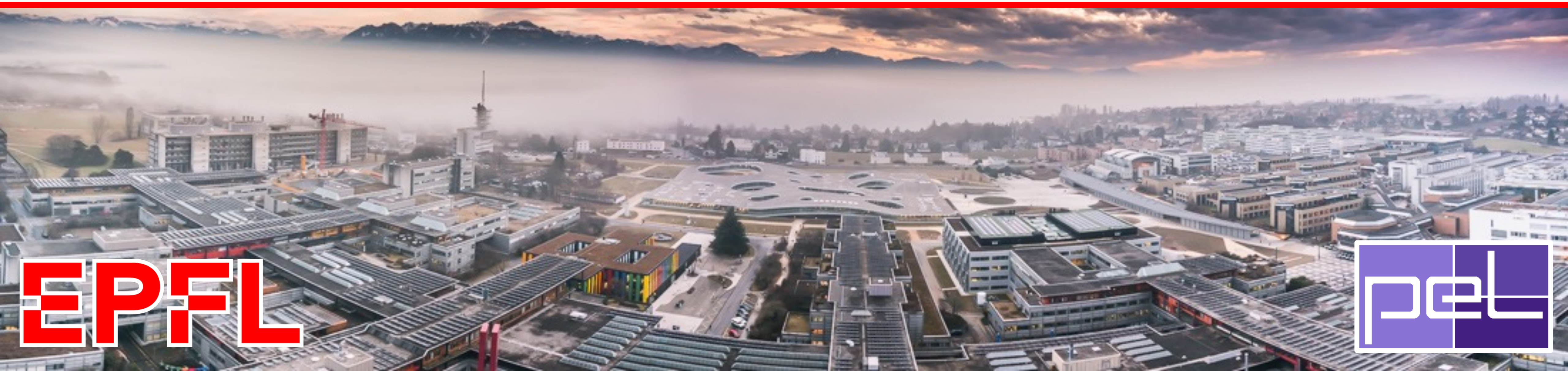
EE-565 – W6
AC MACHINE WINDINGS

Prof. D. Dujic
Power Electronics Laboratory
EPFL
Switzerland



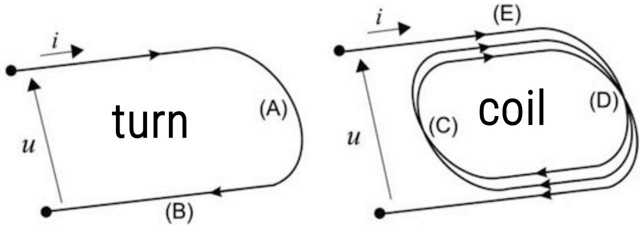
MMF DISTRIBUTION OF A SINGLE-PHASE AC WINDING

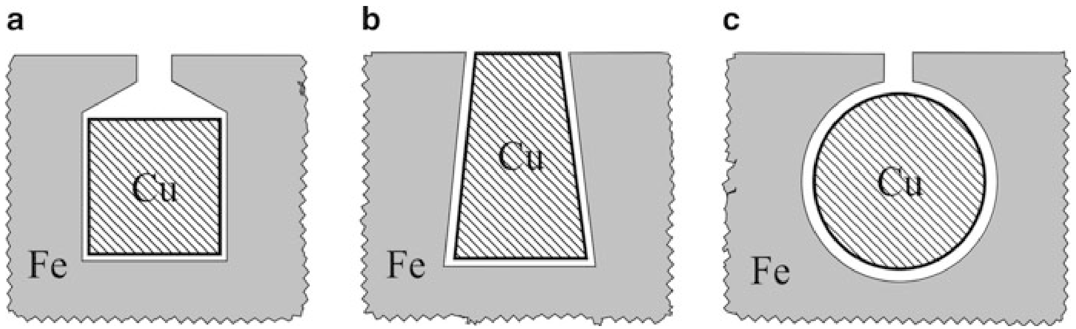
Fundamentals of MMF distribution analysis

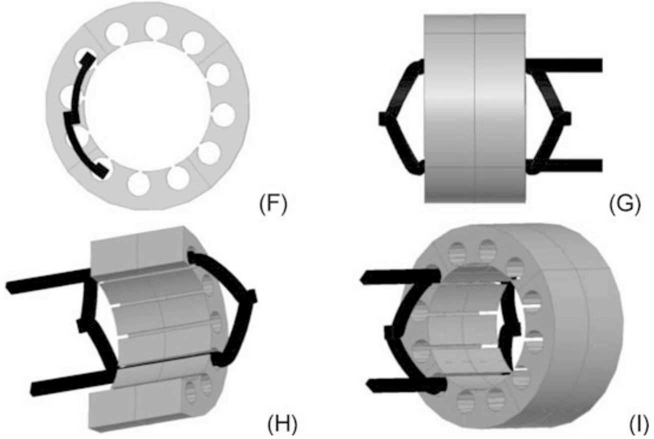
SLOTS IN MAGNETIC CIRCUITS

Magnetic circuits of the stator and rotor are made of iron sheets

- iron sheets (SiFe) are insulated from each other and have **slots** to host windings
- windings are also insulated from each other
- the slots can be of different cross section, depending on the design needs
- the slots usually host more than one conductor (not necessarily of the same winding)
- **Tooth** is the part of the magnetic circuit between neighboring slots
- one **turn** is obtained by a series connection of conductors placed in different slots
- several turns may reside in the same pair of slots, creating a **coil**



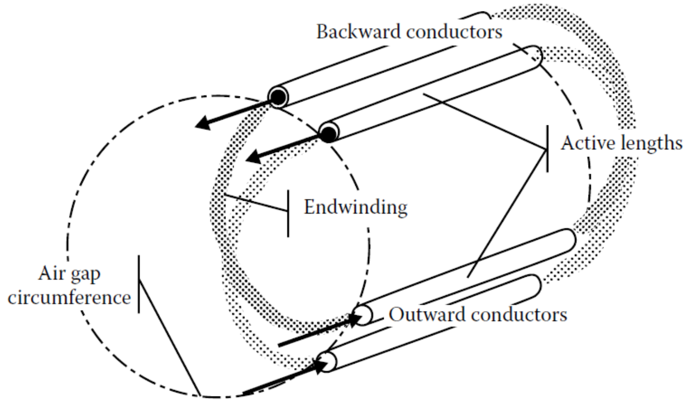


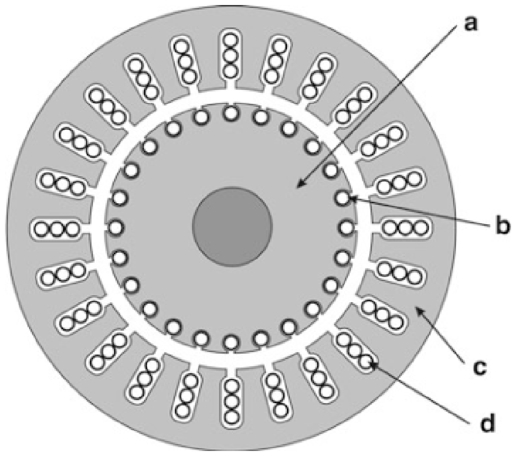


SIMPLIFYING ASSUMPTIONS IN THE MODELING

Some simplifying assumptions are done in the modeling of the AC windings

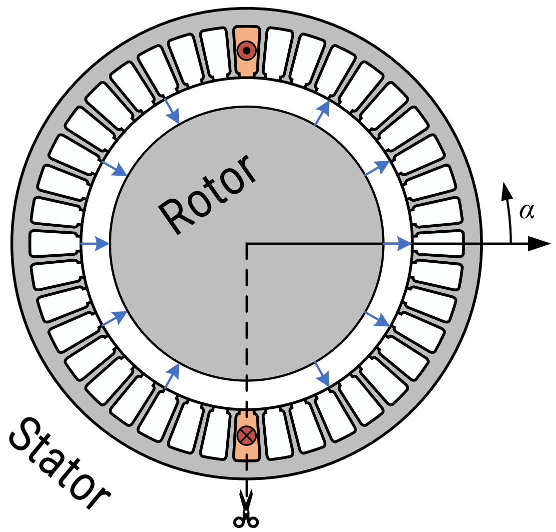
- **The head connections (end-winding) do not participate in the electromagnetic energy conversion**
  - The analysis is carried out only in the active length
- **The magnetic phenomena are identical along the machine axis**
  - It is sufficient to analyze a single 2D slice of the machine
- **The lamination permeability of the iron is infinite**
  - The magnetic flux and the magnetomotive force (MMF) are only present in the air-gap
- **The slot-opening width is infinitesimal**
  - All the conductors in one slot are analyzed as a single dot-like conductor positioned in the center
- **The axial and tangential components of the magnetic flux field in the air-gap are negligible**
  - Only the radial component is present
- **The radial component of the field does not depend on the radius**
  - It is only a function of the angular position
- **The leakage flux of each winding is modeled independently and externally**
  - No mutual leakage flux, all the field at the air-gap goes from stator to rotor
- **The air-gap is of constant width (only for Isotropic Machine Construction)**
  - For Anisotropic machine construction the model must be revised (air-gap permeance function)

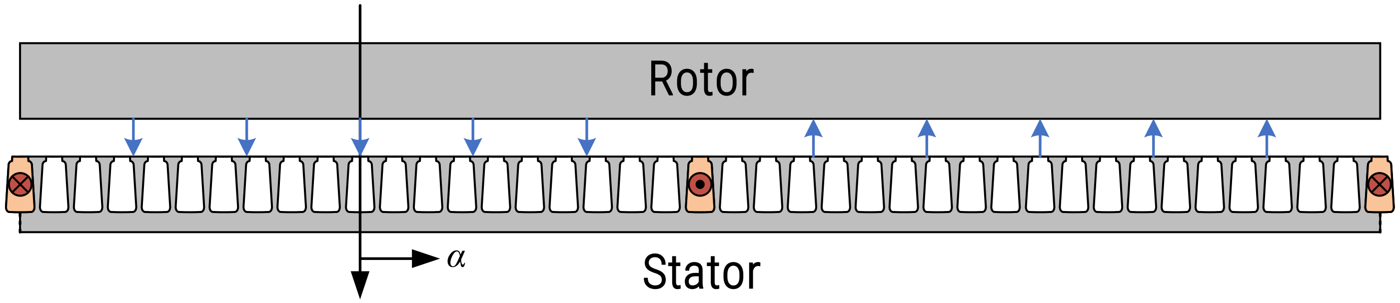




RECTIFIED VIEW OF THE MACHINE

- Consider only a coil on the stator made of Z_f conductors in series ($Z_f/2$ in each slot)
- The conductors are diametrally positioned (**full-pitch**)
- We can set a reference position and introduce an **angular coordinate system** (α)
- For ease in representation, a **rectified view** of the machine is adopted
- A positive direction is defined – from the rotor to the stator





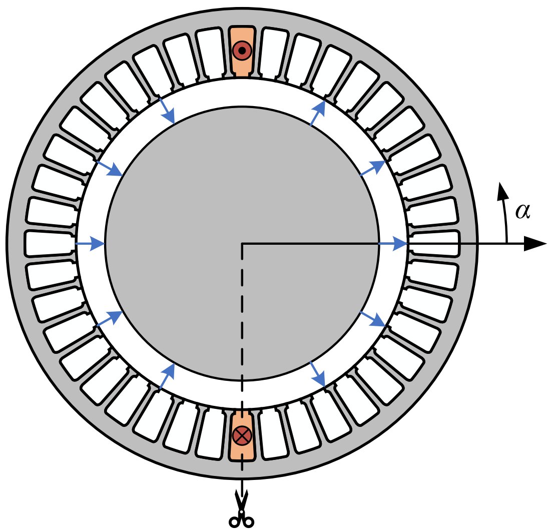
MAGNETOMOTIVE FORCE

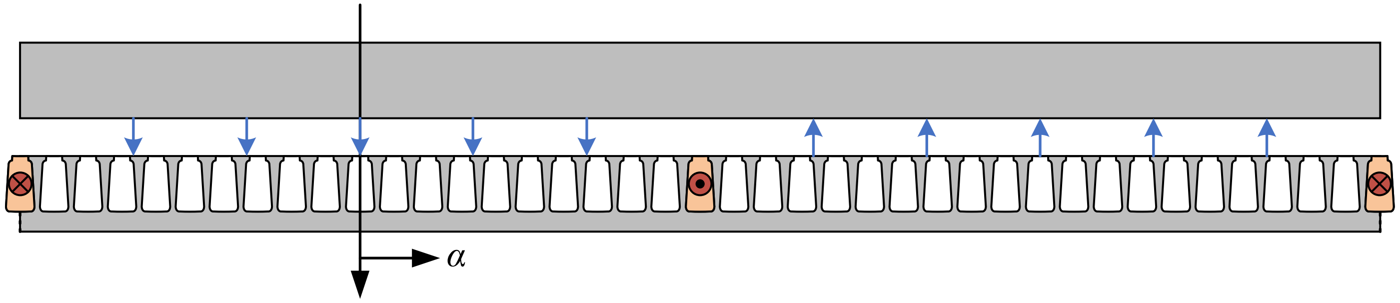
- From the Ampere-Maxwell Equation

$$\underbrace{\oint_\gamma \vec{H}\cdot\hat{t}}_{\text{Tangential Magnetic Field}}\,\mathrm{d}l = \underbrace{I_\gamma}_{\text{Total current encircled by }\gamma}$$

- The magnetic field is zero in the iron (infinite permeability)
- The product of the magnetic field by the air-gap length is the **magnetomotive force (MMF)**

$$\underbrace{F(\alpha)}_{\text{MMF at a position }\alpha} = \int_{\underbrace{P_0}_{\text{Initial point (at a given angle }\alpha)}}^{\overbrace{P_1}^{\text{Final point (at a given angle }\alpha)}} \vec{H}\cdot\hat{t}\,\mathrm{d}l \approx \underbrace{H(\alpha)}_{\text{Magnetic Field at a position }\alpha}\cdot\overbrace{\delta_t}^{\text{Field line length}}$$

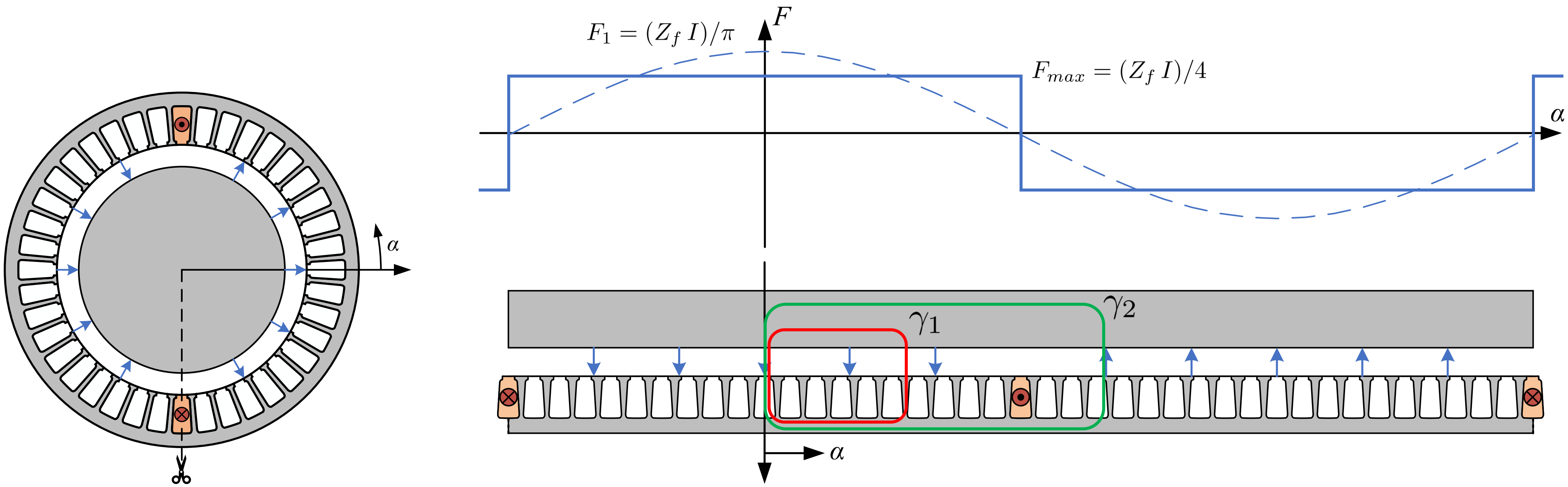




MMF WAVEFORM OF A SINGLE FULL-PITCH COIL

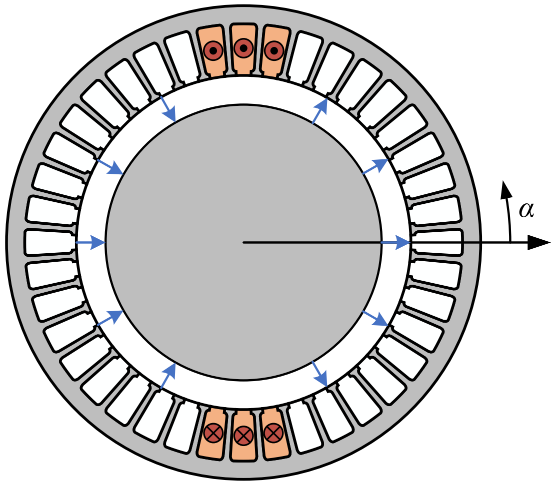
- Considering a curve as in γ_1 $\quad F(0) - F(\alpha) = I_{\gamma_1} = 0$
- Considering a curve as in γ_2 $\quad F(0) - F(\alpha) = I_{\gamma_2} = (Z_f/2) \cdot i$

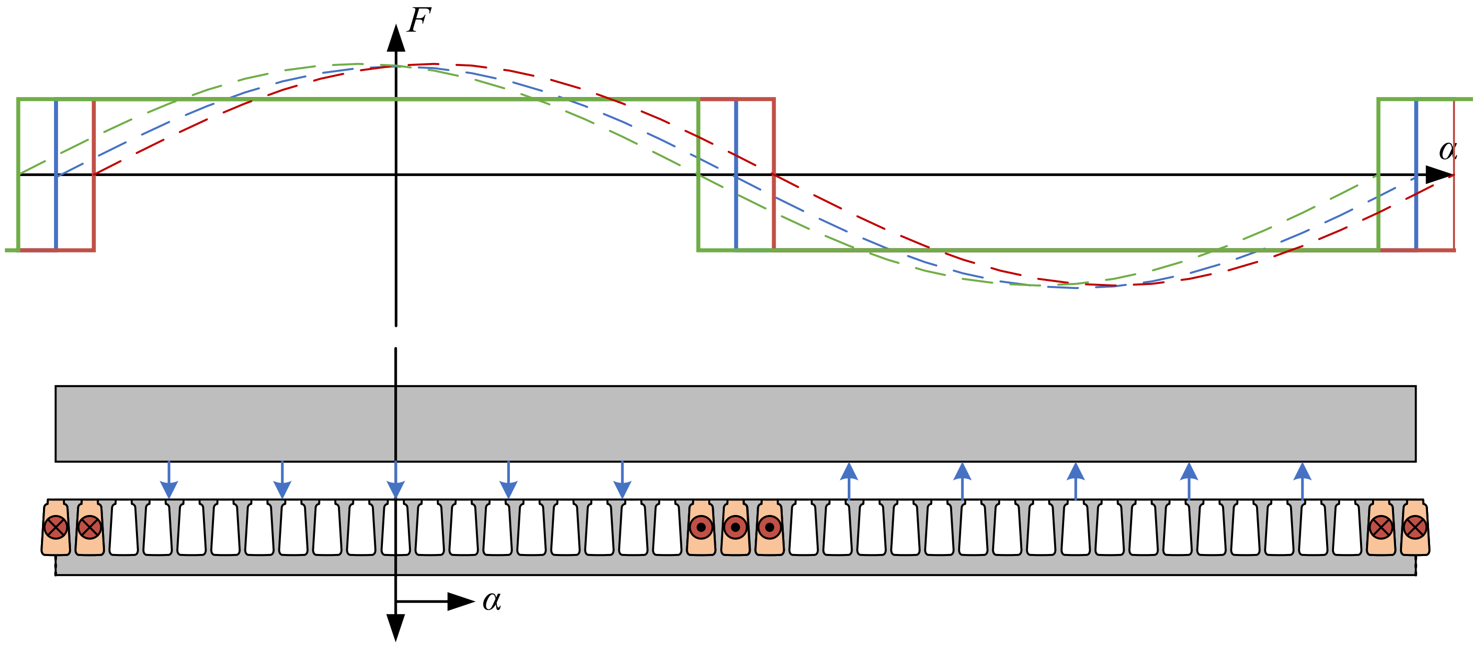
  } ▶ **The MMF distribution is a square-wave**
- **The average value is zero** (because of Gauss's equation on the magnetic flux density field)
- The peak value is: $F_{max} = (Z_f\, I)/4$
- The magnitude of the fundamental (spatial) component is: $F_1 = (Z_f\, I)/\pi$



MMF WAVEFORM OF A SINGLE-PHASE DISTRIBUTED WINDING

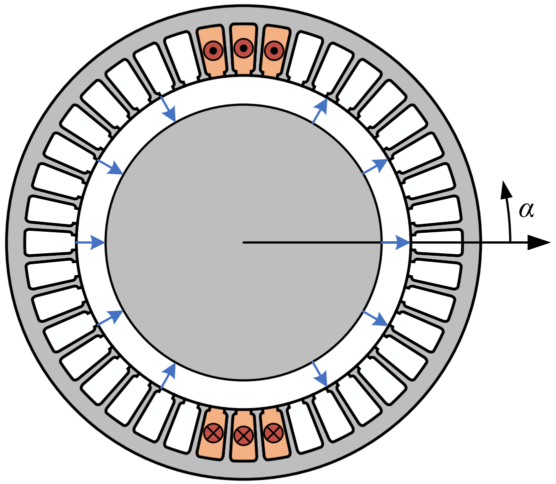
- In a distributed winding, multiple coils are located in different slots
- Their MMF waveforms can be composed together

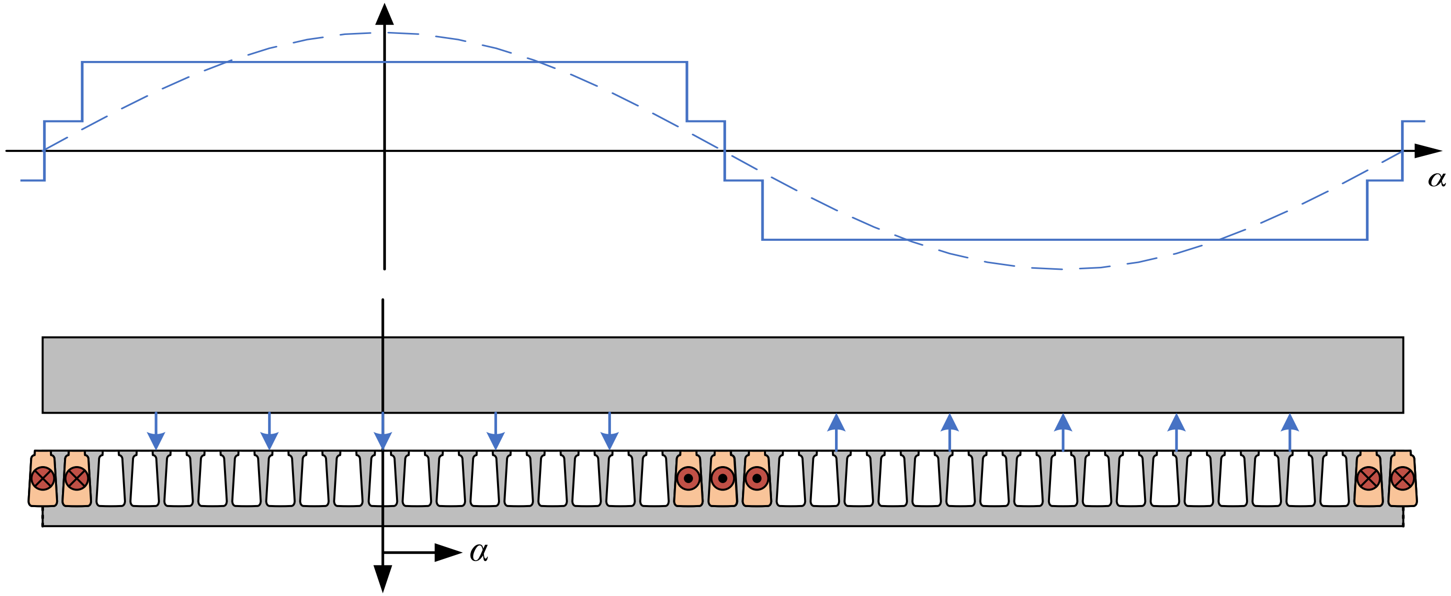




MMF WAVEFORM OF A SINGLE-PHASE DISTRIBUTED WINDING

- In a distributed winding, multiple coils are located in different slots
- Their MMF waveforms can be composed together
- A **staircase MMF waveform** is generated
- The peak value is again: $F_{max} = (Z_f \, I)/4$
- The magnitude of the fundamental (spatial) component is lower than in the concentrated case
- The higher order spatial harmonics can be reduced (the MMF is closer to a sine wave)



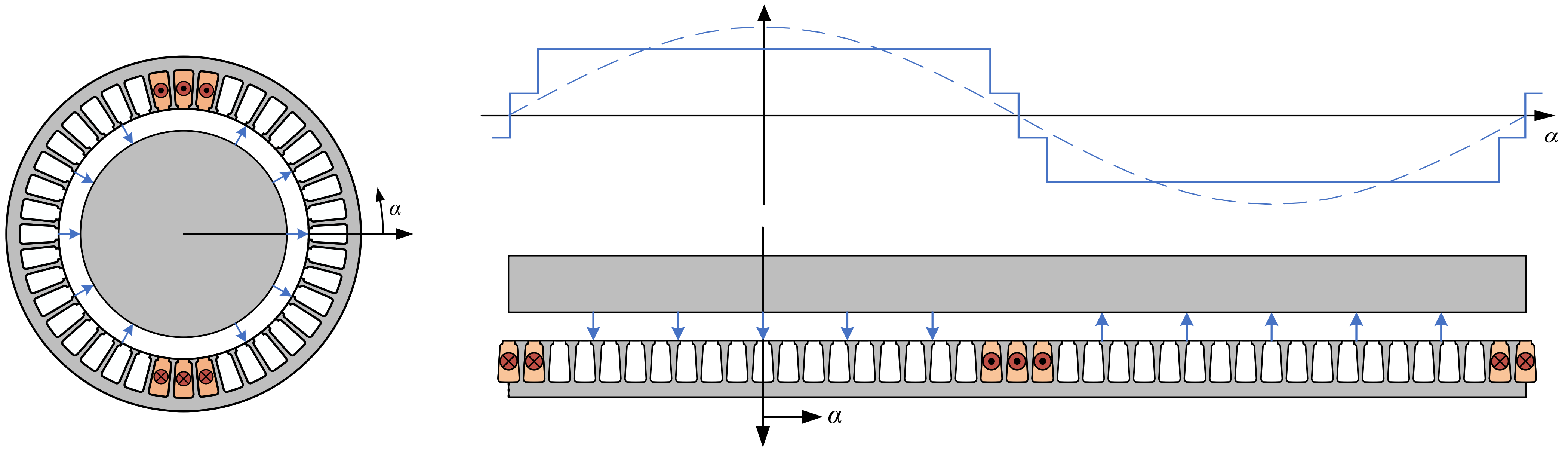


MMF WAVEFORM OF A SINGLE-PHASE DISTRIBUTED WINDING

- The magnitude of the fundamental (spatial) component is lower than in the concentrated case

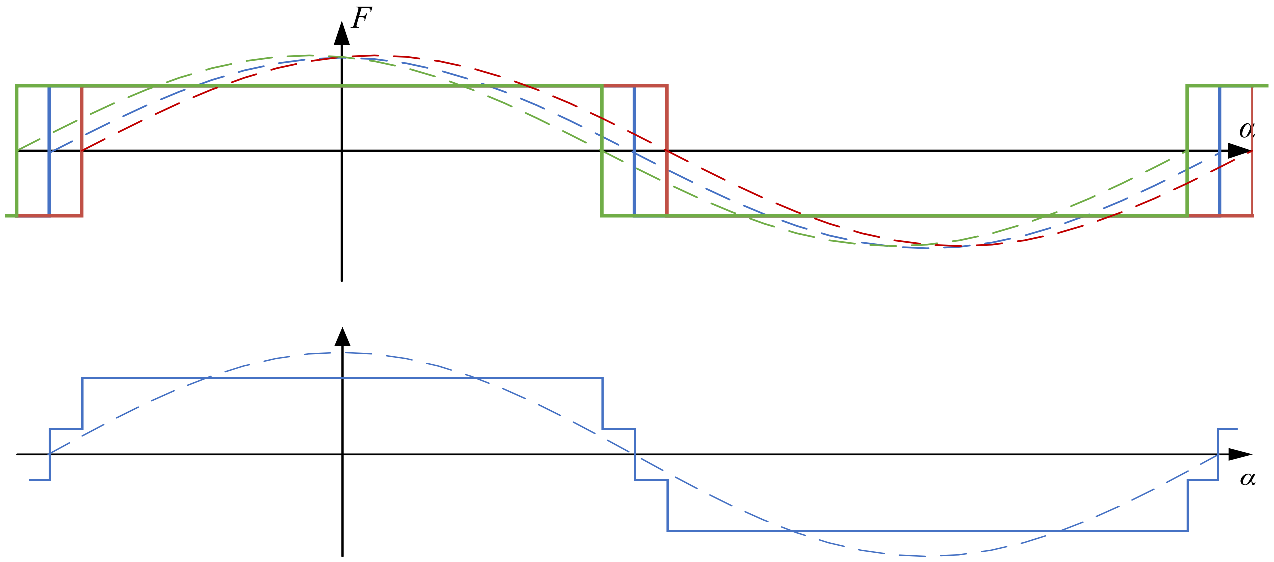
- The decrease of the magnitude is expressed by the **Distribution Coefficient (Belt Factor)**

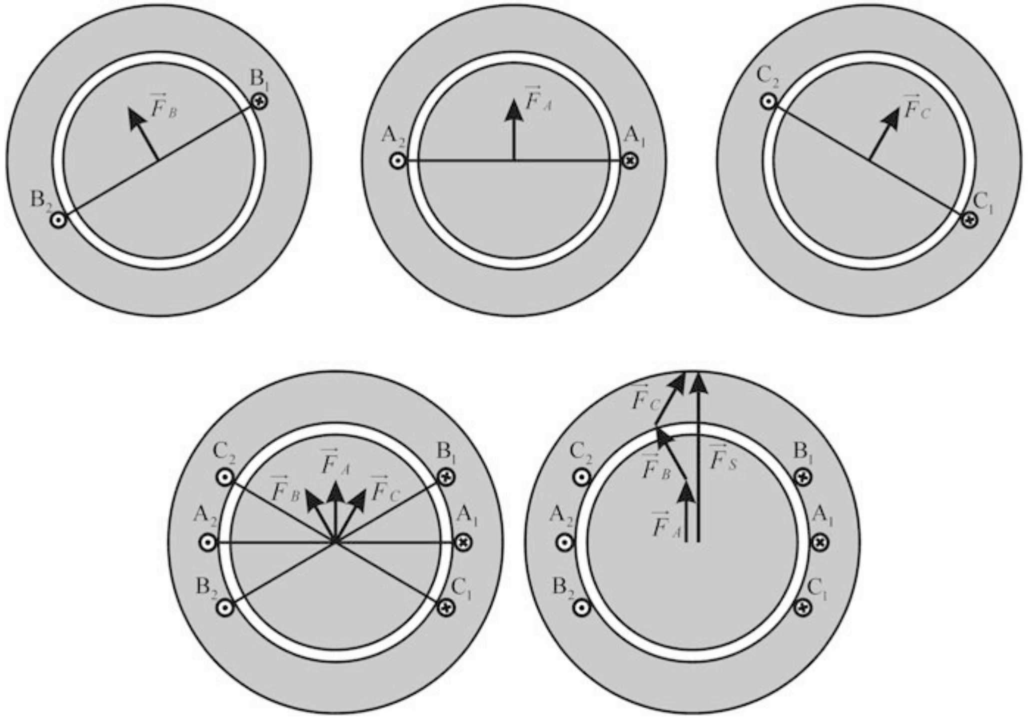
$$F_1 = \underbrace{\frac{Z_f\, I}{\pi}}_{\text{MMF with concentrated winding}} \cdot \frac{\sin(\overbrace{q}^{\text{Number of slots}} \cdot \beta/2)}{q \cdot \sin(\underbrace{\beta}_{\text{Angle between two slots}}/2)} = \frac{Z_f\, I}{\pi} \cdot \underbrace{K_d}_{\text{Distribution Coefficient}}$$



SPACE PHASORS REPRESENTATION

- ▶ The fundamental spatial harmonic of the MMF of each coil is a sine wave with respect to the angle α
- ▶ It can be associated to a complex number identifying magnitude and phase angle
- ▶ This is equivalent to a **Phasor** representation, but applied in space instead of time (**Space Phasor**)
- ▶ Combining different coil can be obtained more easily as vector sum of their space phasors
- ▶ This is closely related to the definition of space vectors for electrical quantities (explained later)

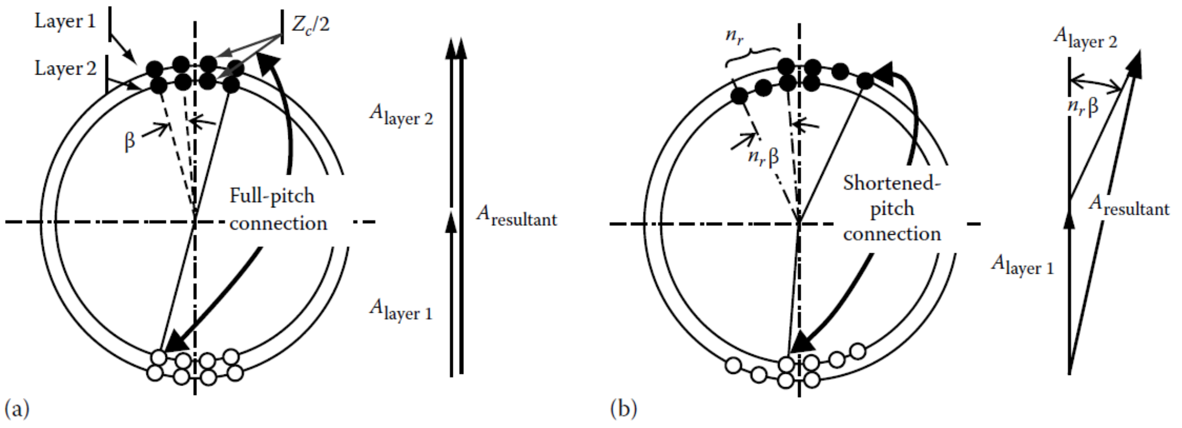


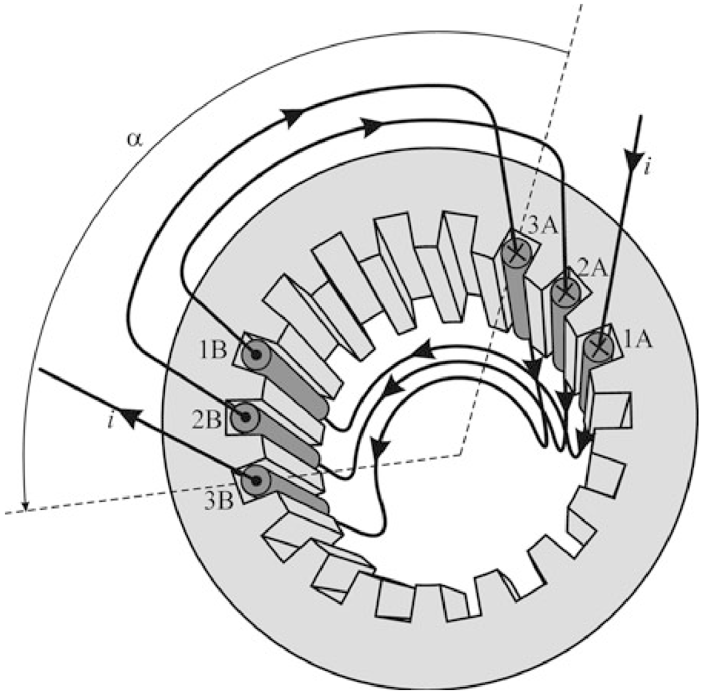


MMF WAVEFORM OF A SHORTENED-PITCH WINDING

- In many cases, the conductors of a coil are not positioned diametrally, but at an **angle less than 180°** (**Shortened Pitch Coil**)
- Additionally, the winding can be divided in **multiple layers** physically positioned in the same slots
- It allows **reducing the length of the head connection** and can be used to **reduce the higher order harmonics in the MMF**
- However, also the fundamental harmonic is reduced compared to the full-pitch case
- The decrease of the magnitude is expressed by the **Shortening Coefficient (Pitch Factor)**

$$\underbrace{F_{1,tot}}_{\text{Total MMF}} = \underbrace{F_{1,layer}}_{\text{MMF of a single layer}} \cdot \cos(\underbrace{n_r}_{\text{Number of slots for shortening}}\ \beta/2) = F_{1,layer} \cdot \underbrace{K_r}_{\text{Shortening coefficient}}$$





WINDING COEFFICIENT AND EQUIVALENT NUMBER OF TURNS

- For the overall MMF waveform, considering distribution and shortening, the magnitude of the fundamental component is identified by a **Winding Coefficient** (product of distribution coefficient and shortening coefficient)

$$\overbrace{F_{1,tot}}^{\text{Fundamental MMF for distributed winding}} = K_a \cdot \underbrace{\frac{\overbrace{Z_f}^{\text{Total number of conductors}} I}{\pi}}_{\text{Fundamental MMF for concentrated winding}} \quad \Rightarrow \quad \underbrace{K_a}_{\text{Winding Coefficient}} = K_r \cdot K_d = \underbrace{\cos\left(n_r \frac{\beta}{2}\right)}_{\text{Effect of shortening}} \cdot \underbrace{\frac{\sin(q \cdot \beta/2)}{q \cdot \sin(\beta/2)}}_{\text{Effect of distribution}}$$

- A winding can be identified by an **Equivalent Number of Turns**, that **generates only the sinusoidal component of the MMF**

$$\overbrace{F_{1,tot}}^{\text{Fundamental MMF for distributed winding}} = \left(K_a \cdot \frac{Z_f}{\pi}\right) \cdot I = \underbrace{N'}_{\text{Equivalent number of turns}} \cdot I \quad \Rightarrow \quad F_1(\alpha) = F_{1,tot} \cdot \cos(\alpha) = N' I \cos(\alpha)$$

- The equivalent number of turns is proportional to the total number of conductors and to the winding coefficient

WINDING POLARITY

- The same considerations can be applied if the winding has multiple identical repetitions along the mechanical periphery
- The number of identical repetitions defines the **Pole Pairs** of the winding P_p
- The equivalent number of turns is proportional to the total number of conductors and to the winding coefficient
- An **electrical angle** is defined as a reference covering only $360°/P_p$ mechanical angles
- The previous definitions are modified as

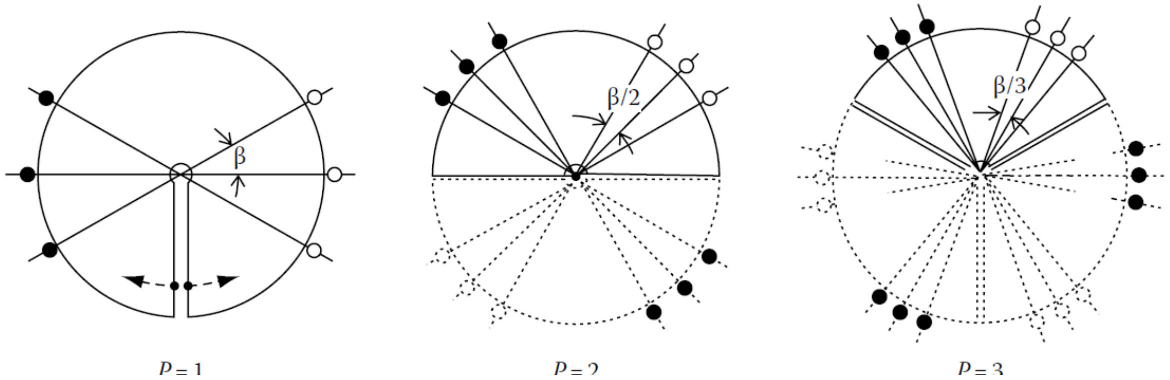
$$\alpha_e = P_p \alpha$$

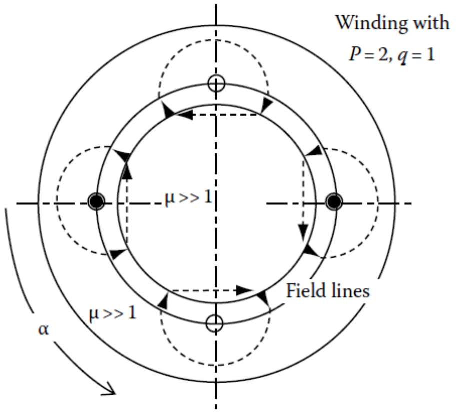
$$\beta_e = P_p \beta$$

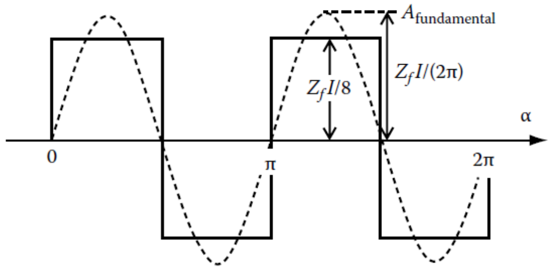
$$F_{1,tot} = K_a \cdot \frac{Z_f}{\boxed{P_p}\pi} \cdot I = N' \cdot I$$

$$K_a = K_r \cdot K_d = \cos\left(n_r \frac{\boxed{\beta_e}}{2}\right) \cdot \frac{\sin(q \cdot \boxed{\beta_e}/2)}{q \cdot \sin(\boxed{\beta_e}/2)}$$

$$F_1(\alpha) = F_{1,tot} \cdot \cos(\boxed{P_p \alpha}) = N' I \cos(\boxed{\alpha_e})$$



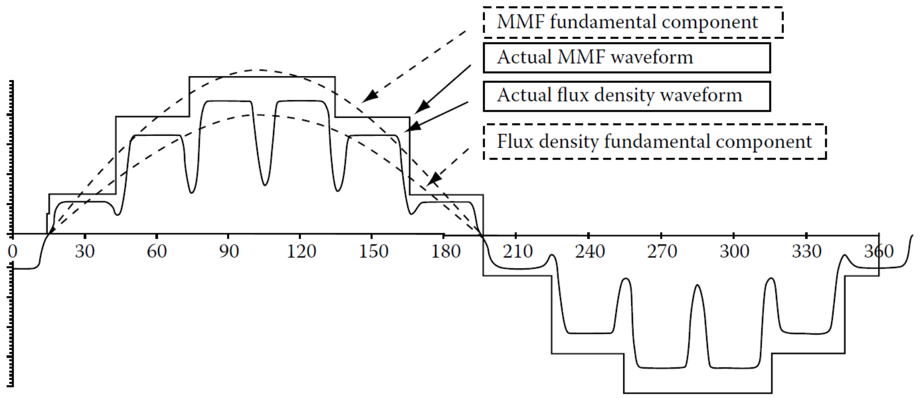




MAGNETIC FLUX DENSITY FIELD DISTRIBUTION

- The magnetomotive force is the magnetic field times the air-gap length
- The distribution of the **magnetic flux density field in the air-gap** can be found as:

$$B(\alpha) = \mu_0 \, H(\alpha) = \mu_0 \frac{F(\alpha)}{\delta_t(\alpha)}$$



- Slotting and magnetic anisotropy can modify the MMF waveform
- For the fundamental component evaluation in isotropic machine designs, an approximate formulation can be used

$$B_1(\alpha) = \mu_0 \, H(\alpha) = \mu_0 \frac{F(\alpha)}{\underbrace{\delta_0}_{\text{Average air-gap length}}}$$

- More advanced expressions can be derived (e.g., Carter coefficients)

MMF DISTRIBUTION OF A THREE-PHASE AC WINDING

Space Vectors and generation of the Rotating Field

MMF WAVEFORMS FOR A THREE-PHASE STATOR WINDING

- In a three-phase winding, the winding of each phase are **shifted between one another by 120° electrically**
- By default, the magnetic axis of the first stator phase is set as a reference to define the angle coordinate system
- The winding distribution of the three phases is identical
- The MMFs generated at the air-gap (considering only the fundamental spatial components) are:

$$F_{s,a}(\alpha_e) = N_s'\, i_{s,a}\ \cos(\alpha_e)$$

$$F_{s,b}(\alpha_e) = N_s'\, i_{s,b}\ \cos\left(\alpha_e - 2\pi/3\right)$$

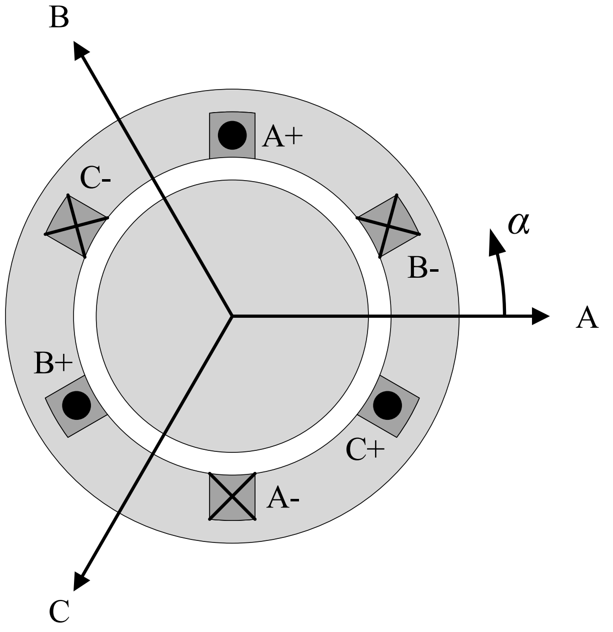
$$F_{s,c}(\alpha_e) = N_s'\, i_{s,c}\ \cos\left(\alpha_e - 4\pi/3\right)$$

- They can be reformulated as:

$$F_{s,a}(\alpha_e) = \mathrm{Re}\{N_s'\, i_{s,a}\ e^{-j\alpha_e}\}$$

$$F_{s,b}(\alpha_e) = \mathrm{Re}\{N_s'\, i_{s,b}\ e^{j2\pi/3}\ e^{-j\alpha_e}\}$$

$$F_{s,c}(\alpha_e) = \mathrm{Re}\{N_s'\, i_{s,c}\ e^{j4\pi/3}\ e^{-j\alpha_e}\}$$

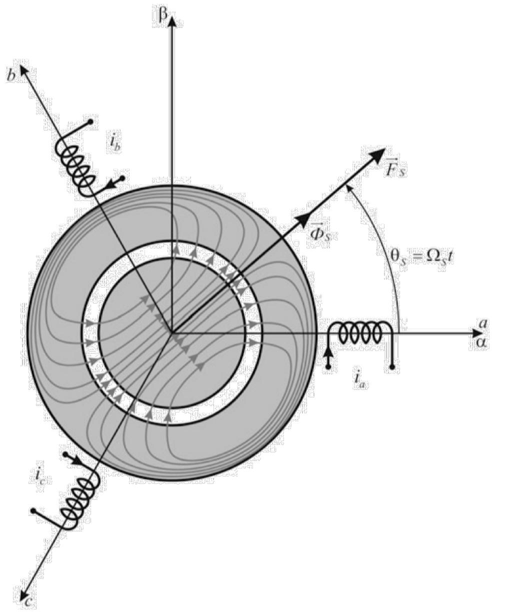


TOTAL MMF WAVEFORM AND SPACE VECTOR OF THE STATOR CURRENTS

- The total MMF waveform produced by the three-phase winding set is given by superposition of the three individual contributions

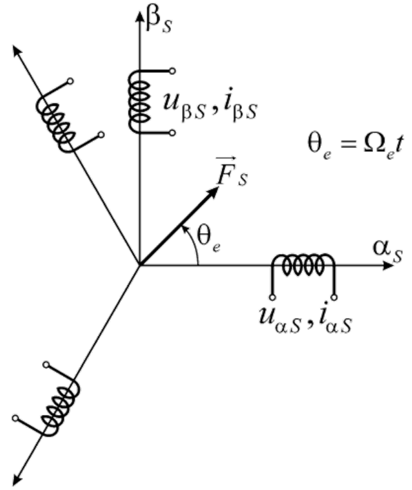
$$F_s(\alpha_e) = F_{s,a}(\alpha_e) + F_{s,b}(\alpha_e) + F_{s,c}(\alpha_e)$$
$$= \mathrm{Re}\{N_s' \underbrace{(i_{s,a} + i_{s,b}\, e^{j2\pi/3} + i_{s,c}\, e^{j4\pi/3})}_{\text{Space vector of the stator currents}} e^{-j\alpha_e}\} = \mathrm{Re}\left\{\frac{3}{2} N_s'\, \underline{i}_s\, e^{-j\alpha_e}\right\}$$

$$\underline{i}_s = i_{s,\alpha} + j i_{s,\beta} = \frac{2}{3}\left(i_{s,a} + i_{s,b}\, e^{j2\pi/3} + i_{s,c}\, e^{j4\pi/3}\right)$$ (Amplitude invariant definition)



- The **Space Vector** of the stator currents **identifies the space phasor of the air-gap MMF**
- The real part (i_α) and the imaginary part (i_β) of the space vector can be interpreted as an **equivalent two-phase system** (with 90° phase shift)
- In matrix formulation we can define the **Clarke's Transformation** as:

$$\begin{bmatrix} i_{s,\alpha} \\ i_{s,\beta} \end{bmatrix} = \frac{2}{3} \cdot \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \cdot \begin{bmatrix} i_{s,a} \\ i_{s,b} \\ i_{s,c} \end{bmatrix}$$

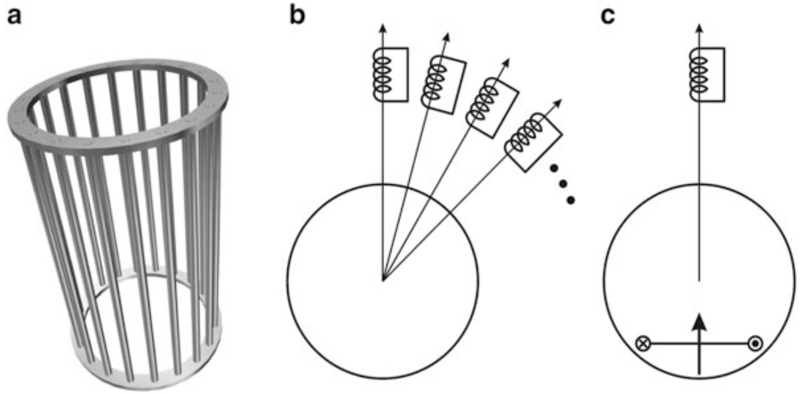


GENERALIZATIONS

- In general, for a symmetrical m-phase system

$$\underline{i} = i_\alpha + j i_\beta = \frac{2}{m}\sum_{k=1}^{m} i_k\, e^{j\alpha_k} \qquad F_s(\alpha) = \mathrm{Re}\left\{\frac{m}{2} N' \,\underline{i}\, e^{-j\alpha_e}\right\}$$

Magnetic axis of the phase k (α_k)



(useful for modeling of multiphase machines and squirrel cage rotors in induction machine)

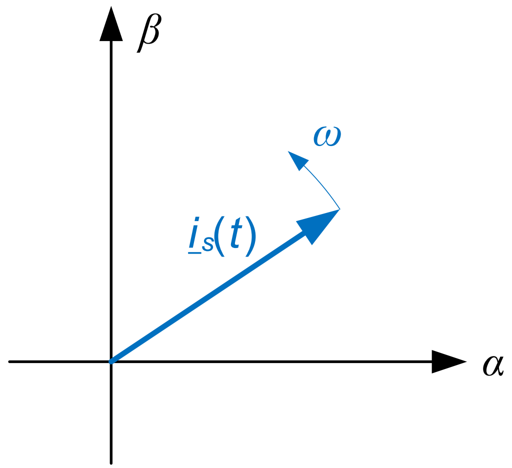
- In a similar way it is possible to define **higher-order space vectors** for **higher-order spatial harmonics** (further on neglected)
  - Higher order harmonics behave as additional windings with $h \cdot P_p$ pole pairs
  - Normally only the odd-order harmonics are present for symmetric windings
  - The equivalent number of turns is different from the fundamental (different winding factor for higher order spatial harmonics)

$$\underline{i}^{\langle h\rangle} = i_{\mathrm{x}}^{\langle h\rangle} + j\, i_{\mathrm{y}}^{\langle h\rangle} = \frac{2}{m}\sum_{k=1}^{m} i_k\, e^{j\,h\,\alpha_k} \qquad F_s^{\langle h\rangle}(\alpha) = \mathrm{Re}\left\{\frac{m}{2} N^{\langle h\rangle\prime}\, \underline{i}^{\langle h\rangle}\, e^{-j\,h\,\alpha_e}\right\}$$

ROTATING FIELD GENERATION

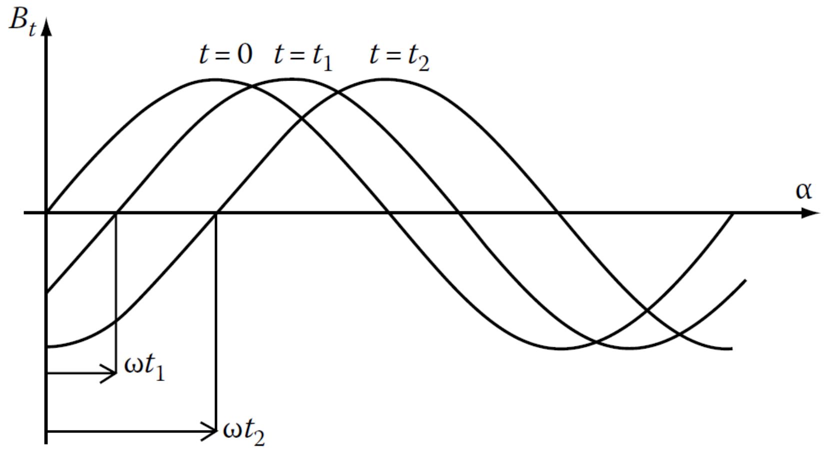
- If the stator currents form a symmetrical sinusoidal three-phase set (in the time domain)

$$i_{s,a}(t) = I_m \cos(\omega t)$$
$$i_{s,b}(t) = I_m \cos\left(\omega t - 2\pi/3\right)$$
$$i_{s,c}(t) = I_m \cos\left(\omega t - 4\pi/3\right)$$



- The space vector of the stator current is a rotating vector with
  - Constant magnitude (I_m)
  - Constant angular speed (ω)

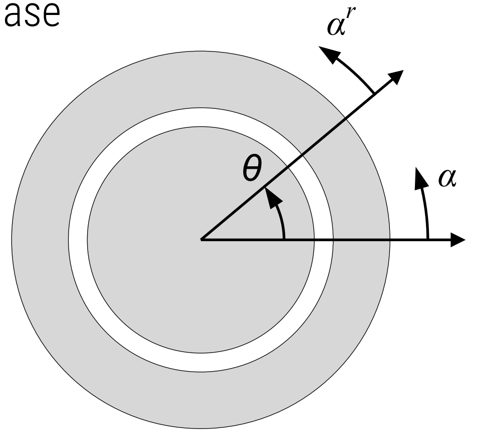
$$\underline{i}_s(t) = \frac{2}{3}\left(i_{s,a} + i_{s,b}\, e^{j2\pi/3} + i_{s,c}\, e^{j4\pi/3}\right) = I_m e^{j\omega t}$$



- **The MMF distribution is a sinusoidal wave** (in space) **that rotates in the air-gap** (in time)
- **The magnetic flux density field follows the same behavior**

MMF WAVEFORMS FOR A THREE-PHASE ROTOR WINDING

- Similar considerations can be applied for the rotor winding
- In this case a different coordinate axis can be defined taking as reference the magnetic axis of the first rotor phase



$$F_r(\alpha_e^r) = \mathrm{Re}\left\{\frac{m_r}{2} N_r' \, \underline{i}_r \, e^{-j\alpha_e^r}\right\}$$

(assuming the same pole pairs periodicity of the stator)

α_e^r: Angle coordinate with respect to the rotor reference

$\underline{i}_r$: Space vector of the rotor currents

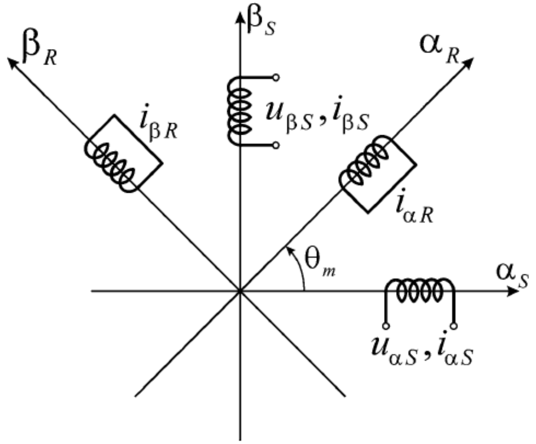
- The two angles systems are linked by the **rotor electrical position** $\theta_e = P_p \, \theta_m$

$$\alpha_e = \alpha_e^r + \theta_e$$

- In the stator coordinate system the rotor MMF distribution is

$$F_r(\alpha_e) = \mathrm{Re}\left\{\frac{m_r}{2} N_r' \, \underline{i}_r \, e^{-j\theta_e} e^{-j\alpha_e}\right\} = \mathrm{Re}\left\{\frac{m_r}{2} N_r' \, \underline{i}_r^{\langle s \rangle} e^{-j\alpha_e}\right\}$$

$\underline{i}_r \, e^{-j\theta_e}$: Space vector of the rotor currents referred to the stator reference frame



INDUCTANCES

Self and Mutual Inductances Expressions

INDUCTANCES IN A TWO WINDING CONFIGURATION

- We want to compute the inductance matrix of a machine
- Consider the case where **only two windings are present** (can be both stator, both rotor or one stator and one rotor winding)
- To keep it general
  - The winding 1 has equivalent number of turns N_1' and magnetic axis at the angle α_1 (can be stationary or time moving)
  - The winding 2 has equivalent number of turns N_2' and magnetic axis at the angle α_2 (can be stationary or time moving)

- The inductances can be found from the electromagnetic energy stored in the machine

$$W_{em} = \frac{1}{2}\boldsymbol{i}^{\mathrm{T}} \cdot \boldsymbol{L} \cdot \boldsymbol{i} = \frac{1}{2}\begin{bmatrix} i_1 & i_2 \end{bmatrix} \cdot \begin{bmatrix} L_{1,1} & L_{1,2} \\ L_{2,1} & L_{2,2} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

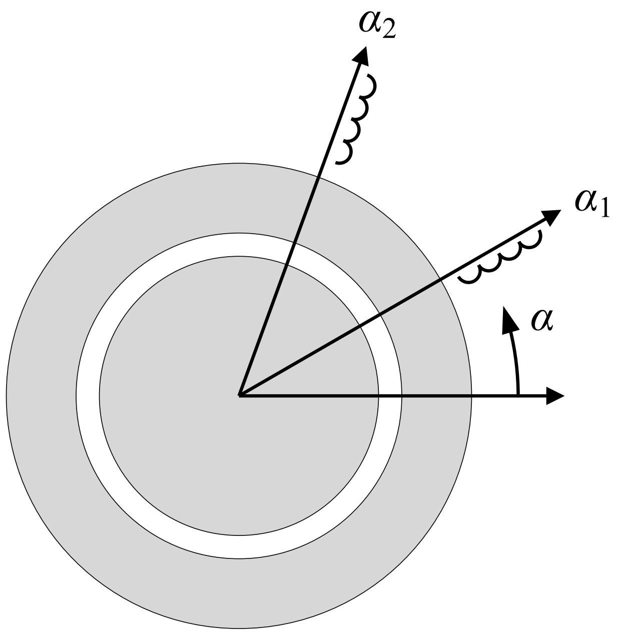
$$= \frac{1}{2}\begin{bmatrix} i_1 & i_2 \end{bmatrix} \cdot \begin{bmatrix} L_1 & L_m \\ L_m & L_2 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$= \underbrace{\frac{1}{2}L_1 i_1^2}_{\text{Energy of the field generated by the winding 1}} + \underbrace{\frac{1}{2}L_2 i_2^2}_{\text{Energy of the field generated by the winding 2}} + \underbrace{L_m i_1 i_2}_{\text{Energy of the fields interaction}}$$

L_1 Self-inductance of winding 1

L_2 Self-inductance of winding 2

L_m Mutual inductance



- We need to find an expression of the energy in the machine based on its parameters

ENERGY IN A TWO WINDING CONFIGURATION

- With a distributed winding configuration and assuming that only the fundamental spatial harmonic at the air-gap is present
- The MMF waveform at the air-gap is:

$$F(\alpha_e) = \underbrace{N_1' i_1 \cos(\alpha_e - \alpha_1)}_{\text{Contribution of Winding 1}} + \underbrace{N_2' i_2 \cos(\alpha_e - \alpha_2)}_{\text{Contribution of Winding 2}}$$

- The magnetic flux density field at the air-gap is:

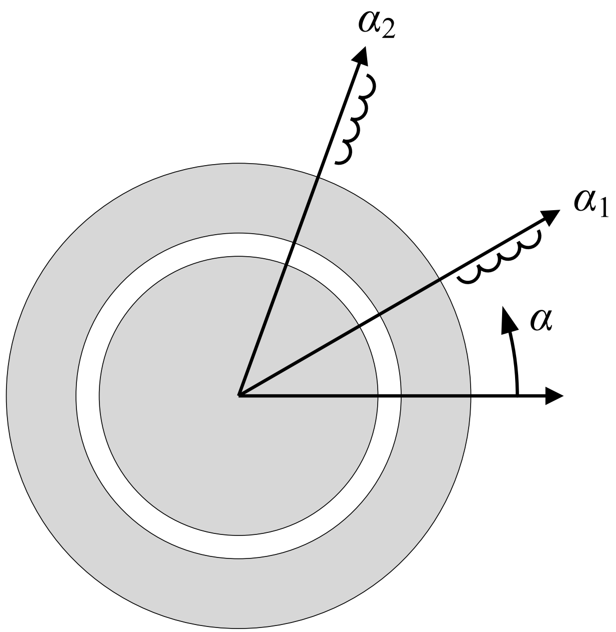
$$B(\alpha_e) = \frac{\mu_0}{\delta_0} F(\alpha_e) = \frac{\mu_0}{\delta_0}\left[N_1' i_1 \cos(\alpha_e - \alpha_1) + N_2' i_2 \cos(\alpha_e - \alpha_2)\right]$$

- The energy is stored only in the air-gap, and the field is only radial

$$W_{em} = \iiint_{V_{em}} \frac{1}{2}\mu H^2 \,\mathrm{d}V = \iiint_{V_{em}} \frac{1}{2}\frac{B^2}{\mu_0}\,\mathrm{d}V$$

$$= \frac{1}{2\mu_0}\, l\, \delta_0\, r \int_0^{2\pi} B^2(\alpha_e)\,\mathrm{d}\alpha_e$$

$$= \frac{1}{2\mu_0}\, l\, \delta_0\, r\, \frac{\mu_0^2}{\delta_0^2} \int_0^{2\pi} \left[N_1' i_1 \cos(\alpha_e - \alpha_1) + N_2' i_2 \cos(\alpha_e - \alpha_2)\right]^2 \mathrm{d}\alpha_e$$



ENERGY IN A TWO WINDING CONFIGURATION

- The energy is stored only in the air-gap, and the field is only radial

$$W_{em} = \frac{1}{2\mu_0}\; l\; \delta_0\; r\; \frac{\mu_0^2}{\delta_0^2} \int_0^{2\pi} \left[N_1' i_1 \cos(\alpha_e - \alpha_1) + N_2' i_2 \cos(\alpha_e - \alpha_2)\right]^2 \mathrm{d}\alpha_e$$

$$= \frac{1}{2}\,\mu_0 \frac{l\, r}{\delta_0} \Bigg[{N_1'}^2\, i_1^2 \underbrace{\int_0^{2\pi} \cos^2(\alpha_e - \alpha_1)\, \mathrm{d}\alpha_e}_{=\pi} + \cdots$$

$$\cdots + {N_2'}^2\, i_2^2 \underbrace{\int_0^{2\pi} \cos^2(\alpha_e - \alpha_2)\, \mathrm{d}\alpha_e}_{=\pi} + \cdots$$

$$\cdots + 2 N_1' N_2'\, i_1\, i_2 \underbrace{\int_0^{2\pi} \cos(\alpha_e - \alpha_1)\, \cos(\alpha_e - \alpha_2)\, \mathrm{d}\alpha_e}_{= \pi \cdot \cos(\alpha_1 - \alpha_2)} \Bigg]$$

$$= \frac{1}{2} \underbrace{\left[\mu_0 \frac{l\, r\, \pi}{\delta_0} {N_1'}^2\right]}_{L_1} \cdot i_1^2 + \frac{1}{2} \underbrace{\left[\mu_0 \frac{l\, r\, \pi}{\delta_0} {N_2'}^2\right]}_{L_2} \cdot i_2^2 + \underbrace{\left[\mu_0 \frac{l\, r\, \pi}{\delta_0} N_1' N_2'\, \cos(\alpha_1 - \alpha_2)\right]}_{L_m} \cdot i_1\, i_2$$

L_1: Self-inductance of winding 1

L_2: Self-inductance of winding 2

L_m: Mutual inductance

INDUCTANCES OF TWO DISTRIBUTED WINDINGS

- The expression of the self-inductances and of the mutual inductances has been found
- They can all be expressed in terms of the **equivalent number of turns** and of a **common magnetic reluctance**

$$L_1 = \mu_0 \frac{l\,r\,\pi}{\delta_0} {N_1'}^2 = \frac{{N_1'}^2}{R_{m,eq}}$$

$$L_2 = \mu_0 \frac{l\,r\,\pi}{\delta_0} {N_2'}^2 = \frac{{N_2'}^2}{R_{m,eq}}$$

$$L_m = \mu_0 \frac{l\,r\,\pi}{\delta_0} N_1' N_2' \cos(\alpha_1 - \alpha_2) = \frac{N_1' N_2'}{R_{m,eq}} \cos(\alpha_1 - \alpha_2)$$

$$R_{m,eq} = \frac{1}{\mu_0} \underbrace{\frac{\delta_0}{S_{coil}}}$$

Equivalent Magnetic Reluctance (Magnetic Resistance)

Surface area of a coil on the air-gap

$$S_{coil} = l\,r\,\pi$$

- **The mutual inductance depends on the angle between the magnetic axes of the two windings**
  - It can assume both positive and negative sign
  - It is zero when the magnetic axes are orthogonal (decoupled windings)
  - It is maximum when the magnetic axes are parallel
- For stator-stator and rotor-rotor interactions, the phase displacement between two magnetic axes is time invariant
- For stator-rotor interaction, the phase displacement between two magnetic axes depends on the rotor position

SUMMARY

AC Machine Windings

MMF WAVEFORM OF DISTRIBUTED WINDINGS

- **MMF Waveform of a single distributed winding** (Sinusoidal MMF approximation)

$$F_k(\alpha_e) = \underbrace{N_k'}_{\text{Equivalent number of turns}} \overbrace{i_k}^{\text{Phase Current}} \cos(\alpha_e - \underbrace{\alpha_k}_{\text{Magnetic Axis}})$$

$$N' = \underbrace{K_a}_{\text{Winding Coefficient}} \cdot \overbrace{Z_f}^{\text{Total number of series conductors}} / (\pi \underbrace{P_p}_{\text{Pole Pairs}})$$

- **MMF Waveform of a symmetrical set of distributed windings**

$$F_s(\alpha_e) = \mathrm{Re}\left\{ \frac{3}{2} N_s' \underbrace{\underline{i}_s}_{\text{Space Vector of the Stator Currents}} e^{-j\alpha_e} \right\}$$

$$\underbrace{\underline{i}_s}_{\text{In the Stator Reference Frame}} = i_{s,\alpha} + j i_{s,\beta} = \frac{2}{3}\left(i_{s,a} + i_{s,b}\, e^{j2\pi/3} + i_{s,c}\, e^{j4\pi/3}\right)$$

$$F_r(\alpha_e) = \mathrm{Re}\left\{ \frac{m_r}{2} N_r' \underbrace{\underline{i}_r}_{\text{Space Vector of the Rotor Currents}} \overbrace{e^{-j\theta_e}}^{\text{Change of reference frame from rotor to stator}} e^{-j\alpha_e} \right\}$$

$$\underbrace{\underline{i}_r}_{\text{In the Rotor Reference Frame}} = i_{r,\alpha} + j i_{r,\beta} = \frac{2}{\underbrace{m_r}_{\text{Number of Phases}}} \sum_{k=1}^{m_r} i_{r,k}\, \overbrace{e^{j\alpha_{r,k}}}^{\text{Position of the magnetic axis of the k-th winding}}$$

INDUCTANCES OF TWO DISTRIBUTED WINDINGS

- **Inductances of two distributed windings** (for constant air-gap)

$$L_1 = \frac{{N_1'}^2}{R_{m,eq}}$$ Self Inductance of Winding 1

$$L_2 = \frac{{N_2'}^2}{R_{m,eq}}$$ Self Inductance of Winding 2

$$L_m = \frac{N_1' \, N_2'}{R_{m,eq}} \cos(\underbrace{\alpha_1 - \alpha_2}_{})$$ Mutual Inductance bewteen the Windings 1 and 2

$\alpha_1 - \alpha_2$: Phase displacement of the magnetic axes (can be time-varying)

$$R_{m,eq} = \frac{1}{\mu_0} \frac{\delta_0}{\underbrace{S_{coil}}_{}}$$ Equivalent Magnetic Reluctance (Magnetic Resistance)

S_{coil}: Surface area of a coil on the air-gap

$$S_{coil} = l \, r \, \pi$$